Understanding of Experimental Data via Multiple Statistical Methods

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Abstract: *This paper describes the possible solutions of non-linear multivariable by using one and two way ANOVA (analysis of variance), linear regression analysis, and non linear regression analysis. Statistical techniques are used to explore better analysis techniques and improve the laser cutting quality by reducing process variations due to inner array (controllable process parameters).*

The problem has already been solved by Taguchi-neural network method using one way ANOVA (analysis of variance and neural networks) in MS thesis. Orthogonal array used in Taguchi method is a very useful technique to reduce the time and cost of the experiment. The data set is very small in this method which causes difficulties in analysis and learning of models. In classification problems decision tree is a very useful technique but it does not predict better results due to small size of data. The results of neural network are encouraging. Taguchi method is normally optimizing input process parameters for single characteristic [25]. In the process industry most of the time needs to improve multiple quality parameters. Two way ANOVA and multi-regression analysis definitely give more detailed picture of problem and the interaction between different variables.

Keywords: *Analysis of variance, Design of experiment, Orthogonal array, Kerf Width, Regression*

INTRODUCTION

In this Independent study, experimental analysis has been carried out to seek the optimum combination (laser power, cutting speed, assist gas pressure and standoff distance) of input parameters in laser cutting process in order to improve the laser cutting quality on some non metallic materials like Urea formaldehyde (Melamine), polystyrene foam and Plywood laminated Melamine etc. The aim of doing this study is to apply different statistical techniques on the same data. Explain analysis and also compare these statistical techniques to understand different statistical techniques and their benefits, disadvantages and errors. The observed values of edge quality, Kerf widths, percent overcut and material removal rate were measured for measuring quality. Taguchi method was used in experimental design using $L_9(3^4)$ orthogonal array for reducing the number of observation sets from 81 to 9. The effect of input parameters on output quality variation was assessed by Taguchi method to determine the optimum input combination.

Different techniques will be applied at this stage to understand these methods and compare them for better application of available tools. The possible methods are:

- One Way ANOVA
- Two Way ANOVA
- Single Variable Linear Regression Analysis
- Multivariable Regression Analysis
- Nonlinear Regression Analysis
- Multivariable nonlinear Regression Analysis

The above problems can be solved by using calculator or excel. But In this study the theme is to learn more and better statistics with its tools and software. Therefore, we will use Excel and SPSS.

Laser cutting is one of the laser beams machining process which cuts and engraves materials through thermal cutting process. The use of laser technology is justified as the quality of the end product is decisively better and the process more reliable even with it's extremely high cost, though cost is constantly falling. Laser applications in plastic materials cutting have increased significantly in industries as it makes possible to achieve a finer quality finished product together with greater process reliability.

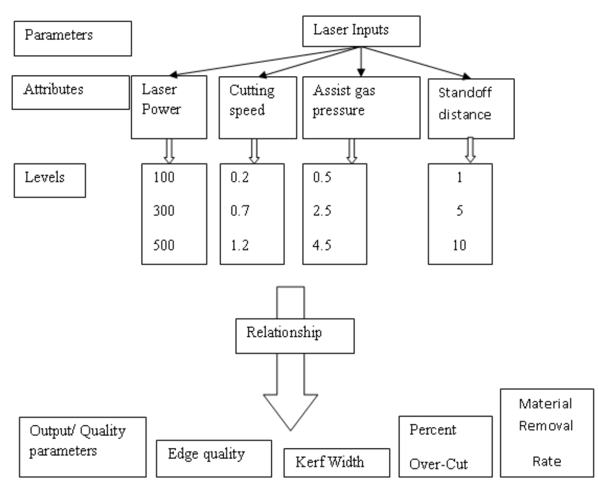


Figure 1: Block Diagram of Input and Output parameters

Figure 1 represents the four controllable input parameters with their value divided into three stages to understand the effect of input parameters on the output parameters [1]. The relationship between the parameters can be found by the modeling of the system by some Mathematical or Statistical method. The model will be used for simulation.

ANALYSIS OF VARIANCE

Table 3: Experimental observations of Kerf Width based on Orthogonal Array

S. No.	A	B	C	D	I	II	III	TPM	NPM
1	100	0.2	0.5	1	1.58	1.62	1.53	1.573	-3.936
2	100	0.7	2.5	5	1.73	1.26	1.48	1.488	-3.454
3	100	1.2	4.5	10	1.66	1.70	1.86	1.738	-4.803
4	300	0.2	2.5	10	1.94	1.89	1.92	1.913	-5.636
5	300	0.7	4.5	1	1.77	1.78	1.99	1.842	-5.304
6	300	1.2	0.5	5	1.66	1.72	1.86	1.742	-4.819
7	500	0.2	4.5	5	2.01	2.04	1.86	1.968	-5.882
8	500	0.7	0.5	10	1.98	1.94	2.29	2.068	-6.312
9	500	1.2	2.5	1	1.79	1.89	2.08	1.920	-5.666

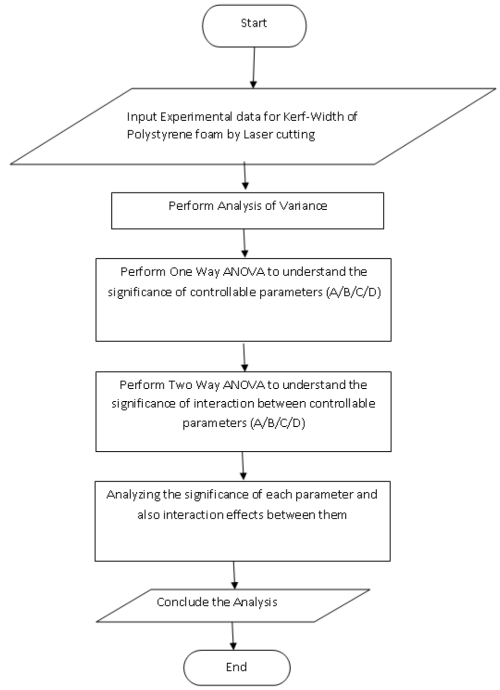
With reference to **Error! Reference source not found.** block diagram, perform One Way ANOVA to understand the significance of controllable parameters (Laser Power (A), Cutting Speed (B), Assist Gas Pressure(C) and Standoff Distance (D). This portion is divided into two parts, with replication and without replication. In the focused experiment polystyrene foam was cut three times with the same input parameter conditions. In the first stage one way ANOVA is performed on MS Excel and SPSS.

In the analysis of variance three assumptions are used:

- The response parameter is normally distributed.
- The variance of the response parameters is same.
- The response parameters are independent.

One Way ANOVA without Replication

In one way ANOVA the effects of input parameter are analyzed one by one on Kerf width quality of Laser Cut. The input parameters were changed in three steps and built an input table based on the research of orthogonal array, which reduced the number of observations and time of experiment [2] hence reducing th



e cost of experiment in the design period.

Figure 2: Block diagram of analysis of variance

Effect of Laser Power on Kerf Width

Table 4: Observations consider Laser Power (A)

100	300	500
1.573	1.913	1.968
1.488	1.842	2.068
1.738	1.742	1.920

Table 5: Summary of descriptive Statistics

Groups	Count	Sum	Average	Variance
100	3	4.800	1.600	0.016
300	3	5.497	1.832	0.007
500	3	5.957	1.986	0.006

Table 6: ANOVA for Laser Power

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	0.226	2	0.113	11.568	0.009	5.143
Within Groups	0.059	6	0.010			
Total	0.285	8				

Sum of square due to treatment SSTR, Mean square due to treatment MSTR are used in between groups and Sum of square due to error SSE, Mean square due to error MSE are used for within groups. The target is smaller average and variance of the Kerf Width the better.

The analysis results show that Kerf Width at 100 watt is better but variance is not small. Table 6 P and F value less than 0.05 means reject null hypothesis Ho i.e. population Means of different groups are not equal. The result shows that Laser Power is significantly participating in the variation of Kerf width quality.

Effect of Cutting Speed on Kerf Width

Table 7: ANOVA for Cutting Speed

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	0.001	2	0.000	0.007	0.993	5.143
Within Groups	0.284	6	0.047			
Total	0.285	8				

The analysis results show that Kerf Width at 0.7m/s is better but variance is not small. Table 6 P and F value accept null hypothesis Ho i.e. the result shows that Cutting Speed is insignificantly participating in the variation of Kerf width quality.

Effect of Assist Gas Pressure on Kerf Width

Table 8: ANOVA for Assist Gas Pressure

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	0.009	2	0.005	0.0997	0.907	5.143
Within Groups	0.276	6	0.046			
Total	0.285	8				

The analysis results show that Kerf Width at 2.5 bar is least mean and variance. Table 6 P and F value accept null hypothesis Ho i.e. the result shows that Assist Gas Pressure is insignificantly participating in the variation of Kerf width quality.

Effect of Standoff Distance on Kerf Width

Table 9: ANOVA for Standoff Distance

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	0.049	2	0.024	0.620	0.569	5.143
Within Groups	0.236	6	0.039			
Total	0.285	8				

The analysis results show that Kerf Width at 5 mm is least mean and variance. Table 6 P and F value accept null hypothesis Ho i.e. the result shows that Standoff Distance is insignificantly participating in the variation of Kerf width quality.

One Way ANOVA with Replication

Effect of Laser Power on Kerf Width

Table 10: Observations consider Laser Power with replication

100	300	500
1.580	1.940	2.010
1.615	1.885	2.040
1.525	1.915	1.855
1.730	1.765	1.975
1.255	1.775	1.940
1.480	1.985	2.290
1.660	1.655	1.790
1.695	1.715	1.890
1.860	1.855	2.080

Table 11: ANOVA for Laser Power

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	0.678	2	0.339	16.066	0.000	3.403
Within Groups	0.507	24	0.021			
Total	1.185	26				

The analysis results show that Kerf Width at 100 watt is least mean and variance. Table 6 P and F value reject null hypothesis Ho i.e. the result shows that Laser Power is significantly participating in the variation of Kerf width quality. The results are improved with replication.

Effect of Cutting Speed on Kerf Width

Table 12: ANOVA for Cutting Speed

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	0.002	2	0.001	0.021	0.979	3.403
Within Groups	1.183	24	0.049			
Total	1.185	26				

The analysis results show that Kerf Width at 0.7m/s is least mean and variance. Table 6 P and F value accept null hypothesis Ho i.e. the result shows that Cutting Speed is insignificantly participating in the variation of Kerf width quality. The results are improved with replication.

Effect of Assist Gas Pressure on Kerf Width

Table 13: ANOVA for Assist Gas Pressure

Source of Variation	SS	Df	MS	F	P-value	F crit
Between Groups	0.027	2	0.014	0.285	0.755	3.403
Within Groups	1.157	24	0.048			
Total	1.185	26				

The analysis results show that Kerf Width at 2.5 bar is least mean and variance. Table 6 P and F value accept null hypothesis Ho i.e. the result shows that Assist Gas Pressure is insignificantly participating in the variation of Kerf width quality. The results are improved with replication.

Effect of Standoff Distance on Kerf Width

Table 14: ANOVA For Standoff Distance

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	0.146	2	0.073	1.691	0.206	3.403
Within Groups	1.039	24	0.043			
Total	1.185	26				

The analysis results show that Kerf Width at 5 mm is least mean and variance. Table 6 P and F value accept null hypothesis Ho i.e. the result shows that Standoff Distance is insignificantly participating in the variation of Kerf width quality.

Two Way ANOVA with Replication

Effect of Laser Power and Cutting Speed on Kerf Width with Replication

With reference to Table 20,"Sample" means Cutting Speed. The F and P value accept null hypothesis Ho i.e. the result shows that Cutting Speed is insignificantly participating in the variation of Kerf width quality. "Columns" mean Laser power. The F and P value reject null hypothesis Ho i.e. the result shows that Laser Power is significantly participating in the variation of Kerf width quality. The reason to perform the analysis is to understand how the Kerf Width behaves when subjected to combination of parameters. "Interaction" stands for effect of Laser Power and Cutting Speed on Kerf Width. The F and P value accept null hypothesis Ho i.e. the result shows that interaction between parameter A & B is insignificantly participating in the variation of Kerf width

Table 15: Interaction between A and B

A/B	100	300	500
0.2	1.580	1.940	2.010
	1.615	1.885	2.040
	1.525	1.915	1.855
0.7	1.730	1.765	1.975
	1.255	1.775	1.940
	1.480	1.985	2.290
1.2	1.660	1.655	1.79
	1.695	1.715	1.89
	1.860	1.855	2.08

Table 16: Interaction between A and B at 0.2

	100/0.2	300/0.2	500/0.2	Total
Count	3	3	3	9
Sum	4.720	5.740	5.905	16.365
Average	1.573	1.913	1.968	1.818
Variance	0.002	0.001	0.010	0.037

Table 17: Interaction between A and B at 0.7

	100/0.7	300/0.7	500/0.7	Total
Count	3	3	3	9
Sum	4.465	5.525	6.205	16.195
Average	1.488	1.842	2.068	1.799
Variance	0.056	0.015	0.037	0.091

Table 18: Interaction between A and B at 1.2

	100/1.2	300/1.2	500/1.2	Total
Count	3	3	3	9
Sum	5.215	5.225	5.760	16.200
Average	1.738	1.742	1.920	1.800
Variance	0.011	0.011	0.022	0.019

Table 19: Total Interaction between A and B

Count	9	9	9
Sum	14.400	16.490	17.870
Average	1.600	1.832	1.986
Variance	0.030	0.012	0.021

Table 20: ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Sample	0.002	2	0.001	0.057	0.945	3.555
Columns	0.678	2	0.339	18.457	$4x10^{-5}$	3.555
Interaction	0.174	4	0.043	2.365	0.092	2.928
Within	0.331	18	0.018			
Total	1.185	26				

Effect of Laser Power and Assist Gas Pressure on Kerf Width with Replication

Table 21: ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Sample	0.027	2	0.014	0.747	0.488	3.555
Columns	0.678	2	0.339	18.457	$4x10^{-5}$	3.555
Interaction	0.148	4	0.037	2.020	0.135	2.928
Within	0.331	18	0.018			
Total	1.185	26				

With reference to Table 20,"Sample" means Assist Gas Pressure. The F and P value accept null hypothesis Ho i.e. the result shows that Assist Gas Pressure is insignificantly participating in the variation of Kerf width quality. "Columns" mean Laser power. The F and P value reject null hypothesis Ho i.e. the result shows that Laser Power is significantly participating in the variation of Kerf width quality. "Interaction" stands for effect of Laser Power and Assist Gas Pressure on Kerf Width. The F and P value accept null hypothesis Ho i.e. the result shows that interaction between parameter A & C is insignificantly participating in the variation of Kerf width.

Effect of Laser Power and Standoff Distance on Kerf Width

Table 22: ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Sample	0.146	2	0.073	3.982	0.037	3.555
Columns	0.678	2	0.339	18.457	0.000	3.555
Interaction	0.030	4	0.007	0.402	0.805	2.928
Within	0.331	18	0.018			
Total	1.185	26				

With reference to Table 20,"Sample" means Standoff Distance. The F and P value reject null hypothesis Ho i.e. the result shows that Standoff Distance is significantly participating in the variation of Kerf width quality.

"Columns" mean Laser power. The F and P value reject null hypothesis Ho i.e. the result shows that Laser Power is significantly participating in the variation of Kerf width quality. The F value of Laser Power is comparably larger than the Standoff Distance. "Interaction" stands for effect of Laser Power and Standoff Distance on Kerf Width. The F and P value accept null hypothesis Ho i.e. the result shows that interaction between parameter A & D is significantly participating in the variation of Kerf width.

Effect of Cutting Speed and Assist Gas Pressure on Kerf Width

Table 23: ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Sample	0.027	2	0.014	0.747	0.488	3.555
Columns	0.002	2	0.001	0.057	0.945	3.555
Interaction	0.825	4	0.206	11.220	$9x10^{-5}$	2.928
Within	0.331	18	0.018			
Total	1.185	26				

With reference to Table 20,"Sample" means Assist Gas Pressure. The F and P value accept null hypothesis Ho i.e. the result shows that Assist Gas Pressure is insignificantly participating in the variation of Kerf width quality. "Columns" mean Cutting Speed. The F and P value accept null hypothesis Ho i.e. the result shows that Cutting Speed is insignificantly participating in the variation of Kerf width quality. "Interaction" stands for effect of Cutting Speed and Assist Gas Pressure on Kerf Width. The F and P value reject null hypothesis Ho i.e. the result shows that interaction between parameter B & C is significantly participating in the variation of Kerf width so will play an important role in the variation of the Kerf Width.

Effect of Cutting Speed and Standoff Distance on Kerf Width

Table 24: ANOVA

Source of Variation	SS	Df	MS	F	P-value	F crit
Sample	0.146	2	0.073	3.982	0.0370	3.555
Columns	0.002	2	0.001	0.057	0.9451	3.555
Interaction	0.706	4	0.176	9.602	0.0002	2.928
Within	0.331	18	0.018			
Total	1.184902	26				

With reference to Table 20,"Sample" means Standoff Distance. The F and P value reject null hypothesis Ho i.e. the result shows that Standoff Distance is significant. "Columns" mean Cutting Speed. The F and P value accept null hypothesis Ho i.e. the result shows that Cutting Speed is insignificantly participating in the variation of Kerf width quality. "Interaction" stands for effect of Cutting Speed and Standoff Distance on Kerf Width. The F and P value reject null hypothesis Ho i.e. the result shows that interaction between parameter B & D is significantly participating in the variation of Kerf width so will play an important role in the variation of the Kerf Width.

Effect of Assist Gas Pressure and Standoff Distance on Kerf Width

Table 25: ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Sample	0.146	2	0.073	3.982	0.0370	3.555
Columns	0.027	2	0.014	0.747	0.4877	3.555
Interaction	0.680	4	0.170	9.257	0.0003	2.928
Within	0.331	18	0.018			
Total	1.185	26				

With reference to Table 20,"Sample" means Standoff Distance. The F and P value reject null hypothesis Ho i.e. the result shows that Standoff Distance is significant. "Columns" mean Assist Gas Pressure. The F and P value accept null hypothesis Ho i.e. the result shows that Assist Gas Pressure is insignificantly participating in the variation of Kerf width quality. "Interaction" stands for effect of Assist Gas Pressure and Standoff Distance on Kerf Width. The F and P value reject null hypothesis Ho i.e. the result shows that interaction between parameter C & D is significantly participating in the variation of Kerf width so will play an important role in the variation of the Kerf Width.

REGRESSION ANALYSIS

In the **Error! Reference source not found.** relationship can be studied by collecting the experimental data. The method of finding the relationship through regression analysis is explained in **Error! Reference source not found.**. After collection of experimental data draw Scatter plot. The scatter plot shows the nature of relation between the variables. They may be Positive linear, negative linear or curvilinear relationships. The flow chart shows that the next step is to calculate coefficient of correlation "r" to see the significance of correlation between the variables. If the value of r is not significant then predicting dependent parameter values by regression equation is a useless practice. If r is significant then determine regression equation by least square method.

$$Y_i = b_1 X_i + b_0 \text{.............................} \textbf{Equation 1}$$

The purpose of regression analysis in this paper is to see the relationship between dependent and independent parameters and predict unknown values as and when required to reduce the time and cost of experimentation and design.

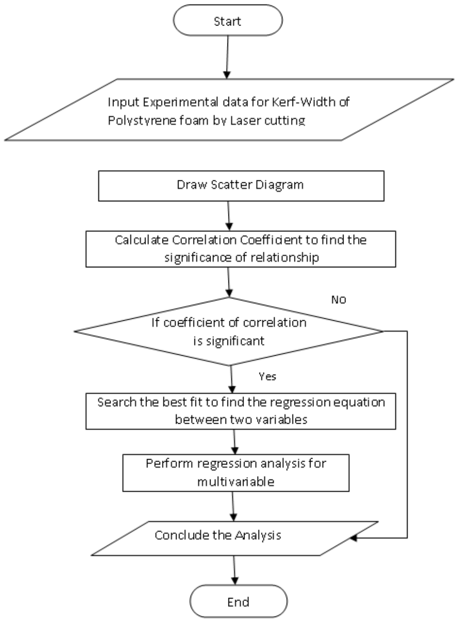


Figure 3: Block Diagram of Regression Analysis

Linear Regression Analysis

Linear Regression of Laser Power and Kerf Width

Scatter plot shows positive 0.885 coefficient of correlation was drawn to observe relationship between Laser Power and Kerf Width. The analysis of Correlation results significant relation between the variable on single tailed test. R^2 shows that the 78.3% variation in Kerf width due to Laser Power that shows it is significant parameter in controlling the quality comparing to other parameters. The adjusted R square is smaller than r square due to small sample size.

Analysis of variance results also show the significant role of Laser Power by F value and significance of F. In linear regression coefficient of line were calculated and T test value and p value which shows that null hypothesis is rejected i.e. population Means are equal. The Laser Power causes significant variation in Kerf width.

Kerf Width residual values in predicted value tables shows that the minimum, maximum, and average percent errors are 5.17%, 21.55%, 12.95% respectively. The values shows that the error is considerably high i.e. more than 5%.

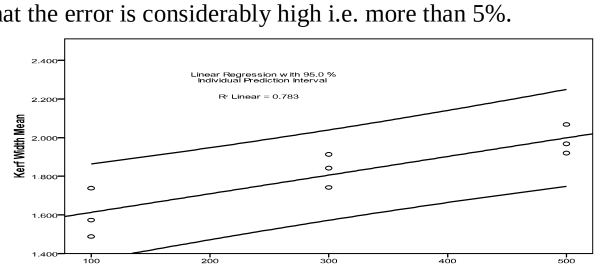


Figure 4: Interactive graph of Laser Power and Kerf Width

Table 26: Correlation

		Kerf Width	Laser Power
Pearson Correlation	Kerf Width Mean	1.000	0.885
	Laser Power	0.885	1.000
Sig. (1-tailed)	Kerf Width Mean	.	0.001
	Laser Power	0.001	.
N	Kerf Width Mean	9	9
	Laser Power	9	9

Table 27: Regression Statistics

R Square	0.783
Adjusted R Square	0.752
Standard Error	0.094
Observations	9

Table 28: ANOVA

	d.f.	SS	MS	F	Significance F
Regression	1	0.223	0.223	25.280	0.0015
Residual	7	0.062	0.009		
Total	8	0.285			

Table 29: Linear regression line of Laser Power

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.517	0.065	23.163	$7x10^{-8}$
(A)	0.001	0.000	5.028	$1x10^{-3}$

Linear Regression of Cutting Speed and Kerf Width

Scatter plot shows negative 0.041 coefficient of correlation was drawn to observe relationship between Cutting Speed and Kerf Width. The analysis of Correlation results insignificant relation between the variable on single tailed test. R^2 shows that the 0.2% variation in Kerf width due to Cutting speed that shows it is not significant parameter in controlling the quality comparing to other parameters. The adjusted R square is smaller than r square due to small sample size.

Analysis of variance results also show the insignificant role of Cutting speed by F value and significance of F. In linear regression coefficient of line were calculated and T test value and p value which shows that null hypothesis is accepted i.e. population Means are equal. The cutting speed causes insignificant variation in Kerf width.

Kerf Width residual values in predicted value tables shows that the minimum, maximum, and average percent errors are 6.2%, 54.83%, 25.77% respectively. The values shows that the error is considerably very high and unacceptable i.e. more than 5%.

Table 30: ANOVA

	Df	SS	MS	F	Significance F
Regression	1	0.001	0.001	0.012	0.9144
Residual	7	0.284	0.041		
Total	8	0.285			

Table 31: Linear regression line of cutting speed

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.819	0.133	13.642	0.0000
(B)	-0.018	0.165	-0.111	0.0914

Linear Regression of Assist Gas pressure and Kerf Width

Scatter plot shows positive 0.126 coefficient of correlation was drawn to observe relationship between Assist Gas Pressure and Kerf Width. The analysis of Correlation results insignificant relation between the variable on single tailed test. R^2 shows that the 1.6% variation in Kerf width due to Assist Gas Pressure that shows it is not significant parameter in controlling the quality comparing to other parameters. The adjusted R square is smaller than r square due to small sample size.

Analysis of variance results also show the insignificant role of Assist Gas Pressure by F value and significance of F. In linear regression coefficient of line were calculated and T test value and p value which shows that null hypothesis is accepted i.e. population Means are equal. The Assist Gas Pressure causes insignificant variation in Kerf width.

Kerf Width residual values in predicted value tables shows that the minimum, maximum, and average percent errors are 1.38%, 54.83%, 25.07% respectively. The values shows that the error is considerably very high and unacceptable i.e. more than 5%.

Table 32: ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.005	0.005	0.113	0.7462
Residual	7	0.280	0.040		
Total	8	0.285			

Table 33: Linear regression line of Assist Gas Pressure

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.772	0.122	14.527	0.0000
(C)	0.014	0.041	0.337	0.746

Linear Regression of Standoff Distance and Kerf Width

Scatter plot shows positive 0.312 coefficient of correlation was drawn to observe relationship between Standoff Distance and Kerf Width. The analysis of Correlation results insignificant relation between the variable on single tailed test. R^2 shows that the 9.8% variation in Kerf width due to Standoff Distance that shows it is significant parameter in controlling the quality comparing to Cutting Speed and Assist Gas Pressure other parameters. The adjusted R square is smaller than r square due to small sample size.

Analysis of variance results also show the insignificant role of Standoff Distance by F value and significance of F.

In linear regression coefficient of line were calculated and T test value and p value which shows that null hypothesis is accepted i.e. population Means are equal. The Standoff Distance causes insignificant variation in Kerf width.

Kerf Width residual values in predicted value tables shows that the minimum, maximum, and average percent errors are 6.38%, 53.97%, 25.94% respectively. The values shows that the error is considerably very high and unacceptable i.e. more than 5%.

Table 34: ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.028	0.028	0.758	0.4128
Residual	7	0.257	0.037		
Total	8	0.285			

Table 35: Linear regression line of Laser Power

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.725	0.112	15.350	0.000
(D)	0.015	0.017	0.871	0.413

Multiple linear Regression

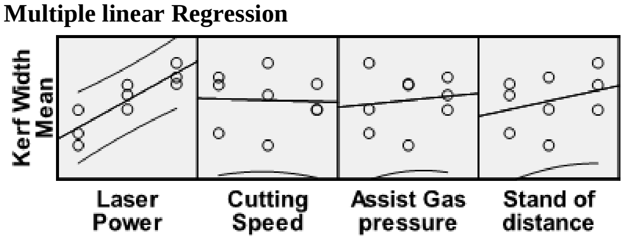


Figure 5: Comprehensive Interactive graph

Table 36: Regression Statistics

Multiple R	0.948
R Square	0.899
Adjusted R Square	0.797
Standard Error	0.085
Observations	9.000

Table 37: ANOVA

	Df	SS	MS	F	Significance F
Regression	4	0.256	0.064	8.858	0.0288
Residual	4	0.029	0.007		
Total	8	0.285			

Table 38: Linear regression of multivariable

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.4147	0.097	14.566	0.0001
(A)	0.0010	0.000	5.557	0.0051
(B)	-0.0183	0.069	-0.264	0.8047
(C)	0.0138	0.017	0.793	0.4723
(D)	0.0151	0.008	1.963	0.1212

Regression analysis between controllable independent parameters (Laser Power, Cutting Speed, Assist Gas Pressure and Standoff Distance) with Kerf Width is explained above.

$\boldsymbol{Y_i = b_1X_{1i} + b_2X_{2i} + b_1X_{3i} + b_1X_{4i} + b_0}$..Equation 2

For the analysis in the beginning Scatter plots are drawn in Figure 4 to observe relationship between the parameters by SPSS software. It shows the linear regression relationship and also drawn 95% confidence interval lines below and above if it is possible with the current scale of the plot. The data points lie in between them. The relation between Kerf Width and input parameters are explained in one graph which shows positive relation of dependent variable with Laser Power positive, Cutting Speed slightly negative, Assist Gas Pressure slightly positive and Standoff Distance is positive.

The calculation of coefficient of correlation shows the value, sign and significance of single tailed test.

- The Laser Power Positive and significant
- The Cutting Speed negative and not significant
- The Assist Gas pressure positive and not significant
- The Standoff Distance positive and not significant

The overall coefficient of correlation is 0.984 showing positive correlation which is more than the effect of any one independent variable effect. R^2 value shows that the variation in Kerf width is 89.9% due to independent parameters which is highly significant parameter in controlling the quality comparing to the variation due to uncontrollable parameters.

Analysis of variance results shows the significant role of independent controllable parameters by F value and significance of F. The results show that null hypothesis is rejected. In linear regression table coefficient of independent parameters are calculated and also T test value and p value of A, B, C and D were calculated. The null hypothesis is accepted except Laser Power (A).

Kerf Width residual values in predicted value tables shows that the minimum, maximum, and average percent errors are 0.71%, 20.66%, 7.99% respectively. The values shows that the error is considerably low i.e. more than 5%. But in the case tolerance margin is little bit wide then the model can be used as an empirical formula.

Nonlinear Regression Analysis

Nonlinear Regression of Laser Power and Kerf Width without Replication

Regression analysis between Laser power and Kerf Width without replication is considered. For the analysis in the beginning Scatter plot is drawn in

Figure **7** shows the nonlinear regression quadratic equation. The coefficient of correlation is 0.891(positive), significant and R^2 value shows that the variation in Kerf width is 79.4% due to Laser Power that shows that it is highly significant parameter in controlling the quality comparing to other parameters.

Y intercept and equation shown on Figure 6. F, T and p value accepted null hypothesis. The Laser Power causes no significant variation in Kerf width in non linear model.

Table 39: Regression data without replication for Laser Power

S. No.	Laser Power A	A^2	Kerf Width Mean
1	100	10000	1.573
2	100	10000	1.488
3	100	10000	1.738
4	300	90000	1.913
5	300	90000	1.842
6	300	90000	1.742
7	500	250000	1.968
8	500	250000	2.068
9	500	250000	1.920

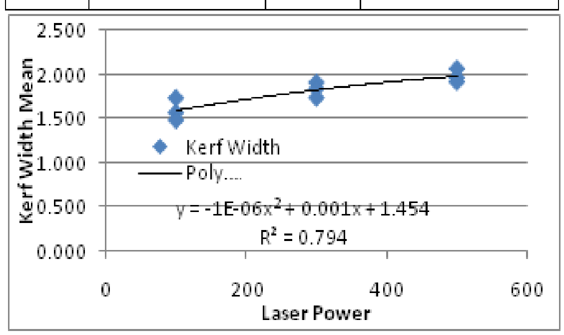


Figure 7: Quadratic graph of Laser Power without replication

Table 40: Regression Statistics

Multiple R	0.891

R Square	0.794
Adjusted R Square	0.725
Standard Error	0.099
Observations	9

Table 41: ANOVA

	d.f.	SS	MS	F	Significance F
Regression	2	0.2261	0.1130	11.5684	0.0087
Residual	6	0.0586	0.0098		
Total	8	0.2847			

Table 42: Nonlinear regression of Laser Power

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.454	0.130	11.154	3.10E-05
A	0.002	0.001	1.457	0.195
A^2	$-9x10^{-7}$	$1x10^{-6}$	-0.564	0.593

Nonlinear Regression of Laser Power and Kerf Width with Replication

Regression analysis between Laser power and Kerf Width with replication is considered. Scatter plot shows the nonlinear regression quadratic equation with coefficient of correlation is positive 0.757, significant and R^2 value shows that the variation in Kerf width is 57.2% due to Laser Power that shows that it is a highly significant parameter in controlling the quality comparing to other parameters. Analysis of variance shows the significant role of Laser power by F value. Regression table calculate the y intercept and equation of in graph. T test value and p value accepted null hypothesis. The Laser Power causes no significant variation in Kerf width.

Kerf Width residual values show that the minimum, maximum, and average percent errors are 1.82%, 59.48%, 55.81% respectively. The values shows that the error is considerably high i.e. more than 50%.

Table 43: ANOVA

	d.f.	SS	MS	F	Significance F
Regression	2	0.678	0.339	16.066	$3x 10^{-5}$
Residual	24	0.507	0.021		
Total	26	1.185			

Table 44: Nonlinear regression of Laser Power

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.4543	0.1106	13.145	$1x10^{-12}$
A	0.0016	0.0009	1.7169	0.0989
A^2	$-9x10^{-7}$	$1x10^{-6}$	-0.665	0.5124

Non Linear Regression of Cutting Speed and Kerf Width

Regression analysis between Cutting Speed and Kerf Width with replication is considered. For the analysis in the beginning Scatter plot is drawn and shows the nonlinear regression quadratic equation with trend line.

The coefficient of correlation is 0.042 (positive) and R^2 value shows that the variation in Kerf width is 0.2% due to Cutting Speed that shows that it is not a significant parameter in controlling the quality comparing to other parameters. Analysis of variance shows the insignificant role of Cutting Speed by F value. T test and p values accepted null hypothesis. The Cutting Speed causes no significant variation in Kerf width.

Kerf Width Predicted residual shows that the minimum, maximum, and average percent errors are 1.724%, 93.84%, 84.75% respectively. The values shows that the error is very high i.e. more than 50%.

Table 45: ANOVA

	d.f.	SS	MS	F	Significance F
Regression	2	0.002	0.001	0.021	0.979
Residual	24	1.183	0.049		
Total	26	1.185			

Table 46: Nonlinear regression of Cutting Speed

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.831	0.145	12.658	$4x10^{-12}$
B	-0.073	0.518	-0.140	0.889
B^2	0.039	0.363	0.107	0.915

Non Linear Regression of Assist Gas Pressure and Kerf Width

Regression analysis between Assist Gas Pressure and Kerf Width with replication is considered. For the analysis in the beginning Scatter plot is drawn and shows the nonlinear regression quadratic equation with trend line.

The coefficient of correlation is 0.152 (positive) and R^2 value shows that the variation in Kerf width is 2.3% due to Assist Gas Pressure that shows that it is not significant parameter in controlling the quality comparing to other parameters. Analysis of variance shows the insignificant role of Assist Gas Pressure by F value. T test and p values accepted null hypothesis. The Cutting Speed causes no significant variation in Kerf width.

Kerf Width Predicted residual shows that the minimum, maximum, and average percent errors are 0.96%, 89.46%, 8.89% respectively. The values shows that the error is very high i.e. more than 50%.

Table 47: ANOVA

	d.f.	SS	MS	F	Significance F
Regression	2	0.027	0.014	0.285	0.755
Residual	24	1.157	0.048		
Total	26	1.185			

Table 48: Nonlinear regression of Assist Gas Pressure

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.815	0.111	16.281	$1x10^{-14}$
C	-0.046	0.115	-0.403	0.691
C^2	0.012	0.022	0.536	0.597

Non Linear Regression of Standoff Distance and Kerf Width

Regression analysis between Standoff Distance and Kerf Width with replication is considered. For the analysis in the beginning Scatter plot is drawn and shows the nonlinear regression quadratic equation with trend line.

The coefficient of correlation is 0.351 (positive) and R^2 value shows that the variation in Kerf width is 12.4% due to Standoff Distance that shows that it is not significant parameter in controlling the quality comparing to other parameters. Analysis of variance shows the insignificant role of Standoff Distance by F value. T test and p values accepted null hypothesis. The Standoff Distance causes no significant variation in Kerf width.

Kerf Width predicted residual shows that the minimum, maximum, and average percent errors are 0.479%, 82.37%, 76.14% respectively. The values shows that the error is very high i.e. more than 50%.

Table 49: ANOVA

	d.f.	SS	MS	F	Significance F
Regression	2	0.146	0.073	1.691	0.206
Residual	24	1.039	0.043		
Total	26	1.185			

Table 50: Nonlinear regression of Standoff Distance

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.815	0.103	17.686	$2x10^{-15}$
D	-0.042	0.049	-0.865	0.395
D^2	0.005	0.004	1.206	0.240

Multiple non linear Regression

Regression analysis between controllable independent parameters (Laser Power, Cutting Speed, Assist Gas Pressure and Standoff Distance) with Kerf Width is explained. For the analysis in the beginning Scatter plots are drawn already. The coefficient of correlation is 0.849 shows positive correlation. R^2 is value shows that the variation in Kerf width is 72.1% due to independent parameters which is highly significant parameter in controlling the quality comparing to the variation due to uncontrollable parameters. Analysis of variance shows the significant role of independent controllable parameters by F value. The results show that null hypothesis is rejected. In Table 29 Nonlinear regression of multivariable calculate the coefficient of quadratic equations and calculate t test and p values for all considered variables in the table which shows that all variable p value more than 0.05 therefore null hypothesis is accepted i.e. unable to explain the variations in dependent variable.

Kerf Width predicted residual shows that the minimum, maximum, and average percent errors are 1.82%, 59.48%, 55.81% respectively. The values shows that the error is considerably high i.e. more than 50%.

Table 51: ANOVA

	d.f.	SS	MS	F	Significance F
Regression	8	0.854	0.107	5.811	0.0010
Residual	18	0.331	0.018		
Total	26	1.185			

Table 52: Nonlinear regression of multivariable

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.498	0.160	9.357	$2.4x10^{-8}$
A	0.002	0.001	1.840	0.082
A^2	$-9x10^{-7}$	$1.3x10^{-6}$	-0.713	0.485
B	-0.073	0.316	-0.230	0.821
B^2	0.039	0.221	0.176	0.863
C	-0.046	0.071	-0.652	0.522
C^2	0.012	0.014	0.868	0.397
D	-0.042	0.032	-1.328	0.201
D^2	0.005	0.003	1.850	0.081

Table 53: ANOVA

	d.f.	SS	MS	F	Significance F
Regression	8	0.854	0.107	5.811	0.0010
Residual	18	0.331	0.018		
Total	26	1.185			

Table 54: Nonlinear regression of multivariable

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.498	0.160	9.357	$2x10^{-8}$
A	0.002	0.001	1.840	0.082
A^2	$-9x10^{-7}$	$1x10^{-6}$	-0.713	0.485
B	-0.073	0.316	-0.230	0.821
B^2	0.039	0.221	0.176	0.863
C	-0.046	0.071	-0.652	0.522
C^2	0.012	0.014	0.868	0.397
D	-0.042	0.032	-1.328	0.201
D^2	0.005	0.003	1.850	0.081

Regression analysis between controllable independent parameters (Laser Power, Cutting Speed, Assist Gas Pressure and Standoff Distance) with Kerf Width is explained above. For the analysis in the beginning Scatter plots are drawn already.

Table 27 shows coefficient of correlation is 0.849 showing positive correlation. R^2 value shows that the variation in

Kerf width is 72.1% due to independent parameters which is highly significant parameter in controlling the quality comparing to the variation due to uncontrollable parameters. The adjusted R square is smaller than r square due to small sample size.

In Table 27 analysis of variance is results also shows the significant role of independent controllable parameters by F value and significance of F. The results shows that null hypothesis is rejected i.e. Means are not equal.

In Table 29 calculate the coefficient of quadratic equations. The hypothesis by T test value and p value of A, B, C and D and their square values are accepted i.e. insignificantly participated in the change in dependent variable. The null hypothesis accepted meaning is that population means are equal. The independent parameters cause insignificant variation in Kerf width.

Kerf Width predicted residual shows that the minimum, maximum, and average percent errors are 1.82%, 59.48%, 55.81% respectively. The values shows that the error is considerably low i.e. more than 50%. But in the case where tolerance margin is more than 50% this empirical formula cannot be used.

DISCUSSION

Discussion on One Way ANOVA

In one way Analysis of Variance results with replication and without replication support the benefit of replication. It means it is better to take replication at least three times for each run of experiment and also better to take five times for bifurcation of error and the variation due to the treatments. The ANOVA table all above cases shows that use of replication improves the F value and p values. It means it can be able to differentiate between controllable and uncontrollable variations.

Discussion on Two Way ANOVA

In Two way Analysis of Variance i.e. interaction shows the Means and Variance due to interaction between the two parameters ignoring other parameters. The significance of Sample, Columns and interaction is measured by significance of F value.

The three of the interaction are not significantly participating in the variation of dependent parameter.

1. Effect of Laser Power and Cutting Speed on Kerf Width with Replication
2. Effect of Laser Power and Assist Gas Pressure on Kerf Width with Replication
3. Effect of Laser Power and Standoff Distance on Kerf Width

These three of the interaction are significantly participating in the variation of dependent parameter.

4. Effect of Cutting Speed and Assist Gas Pressure on Kerf Width
5. Effect of Cutting Speed and Standoff Distance on Kerf Width
6. Effect of Assist Gas Pressure and Standoff Distance on Kerf Width

The interaction 4, 5 and 6 are significant and are ignored so they will consider uncontrollable variables or pooled error.

For better optimization consider all the above interaction in my thesis and previous independent studies and S.B.Tan etal. [1] Will give better results.

Discussion on Linear Regression

In the following table Laser Power is a significant parameter in the variation of Kerf Width and also at a low level so is Standoff Distance. The results verified the analysis of variance technique used in S.B. Tan etal.[1]. The value of R^2 encourage to use the regression modeling techniques but their Maximum residual and average residual errors do not allow to recommend the method to be used. The model can be improved using replication.

Table 55: Summary of linear Regression

	R^2	F sig.	P of t test	Max. error	Average error	Remarks
A	78.3%	0.001	0.001	21.55%	12.95%	Significant
B	0.2%	0.914	0.091	54.83%	25.77%	Insignificant
C	1.6%	0.74	0.746	54.83%	25.07%	Insignificant
D	9.8%	0.412	0.413	53.97%	25.94%	Insignificant

Discussion on Multiple Linear Regression

Regression analysis between controllable independent parameters (Laser Power, Cutting Speed, Assist Gas Pressure and Standoff Distance) with Kerf Width is explained in Section **Error! Reference source not found.**. For the analysis Scatter plots are drawn for linear regression relationship without replication so data points lie in between 95% tolerance limit. The coefficient of correlation and significance are same as linear regression results. The overall coefficient of correlation is improved to 0.984 (positive). R^2 encourage to use the model because Kerf width is 89.9% due to independent parameters which is highly significant parameter in controlling the quality comparing to the variation due to uncontrollable parameters i.e. 10.1 percent only. Analysis of variance results shows the significant role of independent controllable parameters by F, t and P values.

Kerf Width residual predicted value shows that the minimum, maximum, and average percent errors are 0.71%, 20.66%, 7.99% respectively. The values shows that the error is considerably low i.e. around 8%. Therefore, this model can be used to look an idea of the dependent variable trend.

Discussion on Non Linear Regression

Regression analysis between Laser power and Kerf Width without replication is considered in Section **Error! Reference source not found.** Scatter plot is drawn in to observe relationship between the variables. It shows the nonlinear regression quadratic equation with coefficient of correlation is positive 0.891 and coefficient determination R^2 value shows that the variation in Kerf width is 79.4% due to Laser Power that shows that it is a highly significant parameter in controlling the quality compared to other parameters. Laser power effect on Kerf Width significantly even the value of alpha is 1% coefficient of correlation significant relation in single tailed test basis. Analysis of variance shows the significant role of Laser power by F value and significance of F. Regression table calculate the y intercept and equation of trend. T test value and p value accepted null hypothesis that means are equal. The Laser Power causes no significant variation in Kerf width.

Kerf Width residual values in predicted value tables shows that the minimum, maximum, and average percent errors are 1.628%, 23.85%, 11.94% respectively. The values shows that the error is considerably high i.e. more than 5%. The Null hypothesis is rejected in Linear Regression, ANOVA analysis of Laser Power effects on Kerf Width and in multivariable case also rejects the null hypothesis. The data is non linear but regression accept null hypothesis due to data points pattern and error squared in X^2 values.

The value of curve fitting parameters like coefficient of correlation and R^2 are better in nonlinear case but null hypothesis is accepted which is not desirable in this case. The residual values are also not better than the single and multi linear regression.

Non linear with replication

Table 56: Summary of non linear Regression

	% R^2	F sig.	P of t test	Max. error	Average error	H_0
A	79.4	0.009	.19, .59	23.85%	11.94%	Accepted
A	57.2	$3x10^{-5}$	.09, .5	59.48%	55.81%	Accepted
B	0.2	0.979	.89, .92	93.84%	84.75%	Accepted
C	2.3	0.755	.69, .6	89.46%	8.89%	Accepted
D	12.4	0.206	.4, .24	82.37%	76.14%	Accepted

The results of non linear regression show that Laser power is the most important parameter. Its R square value decreases with the replication. The pattern of the data is above or below the last square point. In case of without replication the model touches only one point out of 3 in case of replication it only touches one point out of 9 points. The error is increased due to replication and it will remain increasing if more observations are considered as in factorial design [4]. The null hypothesis H_0 accepted for all variables and their square value which shows that non linear regression with or without replication cannot explain the variations in dependent variables. The residual error due to predicted values increased in non linear case rather than in linear prediction. Nukman Yusoff etal. [3] explain many non linear relations by using only one independent and one dependent variable and keep other constant but in our case three other independent variables are changing along with the considered variable. Therefore this modeling technique cannot be recommended on the basis of the above table results.

Discussion on Multiple Non Linear Regression

Regression analysis between controllable independent parameters with Kerf Width is explained in Section **Error! Reference source not found.** Coefficient of correlation is positive 0.849 and R^2 value shows that the variation in Kerf width is 72.1% due to independent parameters which is highly significant parameter in controlling the quality.

Analysis of variance results also show the significant role of independent controllable parameters by F value and significance of F. The t test results show that null hypothesis is accepted for all parameters. The values shows that the error is considerably high i.e. more than 50%. Therefore, nonlinear regression model is not suitable for our data for simulation and optimization.

CONCLUSION

The null hypothesis is accepted in one way ANOVA except in Laser Power. It shows that Laser Power is the most important factor causes changes in the dependent variable in both case of replication and without replication. It means better to take replication at least three times for each run of experiment and better to take five times for bifurcation of error and the variation due to the treatments i.e. F value. Higher F values small uncontrollable variations using replication gives advantage in ANOVA.

In case of Two Way ANOVA the reason to perform the analysis to understand how the Kerf Width behaves when subjected to combination of two parameters. The null hypothesis is accepted in the three interactions. Therefore these combinations will play an important role in the variation of the dependent parameters:

- Interaction of Cutting Speed and Assist Gas Pressure
- Interaction of Cutting Speed and Standoff Distance
- Interaction of Assist Gas Pressure & Standoff distance

The interaction should be considered to get the better picture of the process optimization. The model of independent study and MS thesis data sets can be improved.

R^2 encourage using the multiple linear regression model because Kerf width is 89.9% due to independent parameters which is highly significant. It can be used in rough modeling, simulation and optimization.

In this study of nonlinear regression has resolved two issues for the given problem.

- The comparison between Linear and non linear model
- Effect of replication on regression analysis

The effect of replication gives worst model due to which predicted values were more non realistic and the average error reaches to 50% which is not acceptable and in with replication is 20%.

The non linear with replication model null hypothesis cannot be rejected even in the case of Laser Power which is sole contributor in the variation of dependent parameter. The overall error increase in case of replication due to increase in the number of observation over and above the fitted points similar to linear regression. The Null hypothesis is rejected in Linear Regression, Multivariable linear Regression and ANOVA analysis of Laser Power effects on Kerf Width but nonlinear regression analysis unable to model the problem.

The following methods were used in previous and current research

- Decision Tree
- One Way ANOVA with pooling techniques
- Simple One Way ANOVA
- Two Way ANOVA
- Linear Regression Analysis
- Non Linear Regression Analysis

The best method was one way ANOVA with pooling but the current research shows that there is one thing was missing that is significantly participating in the variation of dependent parameters i.e. interaction between two independent and one dependent parameters. The discussion of interaction above shows that three combinations of interactions were significantly participating in the variation of Dependent variable. Therefore modeling can be improved with the combination of Treatments and Interaction.

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